BSTA 320 – COMPREHENSIVE EXAM FORMULA SHEET

Decision Under Uncertainty

Maximax

Maximin

Equally Likely (Laplace)

Criterion of Realism (Hurwicz):

$\alpha \times$ (best payoff for an alternative) + $(1 - \alpha) \times$ (worst payoff for the alternative)

Minimax Regret

Decision Making Under Risk

(Outcomes are also known as States of Nature)

Expected Monetary Value

EMV = (payoff of first outcome) × (probability of first outcome)
+ (payoff of second outcome) × (probability of second outcome)
+… + (payoff of last outcome) × (probability of last outcome)

Expected Opportunity Loss

EOL = (regret of first outcome) × (probability of first outcome)
+ (regret of second outcome) × (probability of second outcome)
+… + (regret of last outcome) × (probability of last outcome)

Expected Value with Perfect Information

EVwPI = (best payoff of the first outcome) × (probability of first outcome)
+ (best payoff of the second outcome) × (probability of second outcome)
+… + (best payoff of the last outcome) × (probability of last outcome)

Expected Value of Perfect Information

EVPI = EVwPI – Max EMV

Max EMV: The expected value without information.

Decision Making with Sample Information

Expected Value of Sample Information

$$\text{EVSI} = \begin{pmatrix} \text{expected value of best decision} \\ \text{with sample information,} \\ \text{assuming no cost to gather it} \end{pmatrix} - \begin{pmatrix} \text{expected value of} \\ \text{best decision without} \\ \text{sample information} \end{pmatrix}$$

$$= \text{EVwSI} - \text{Max EMV}$$

Efficiency $= \dfrac{EVSI}{EVPI}$

Compiled by: Math Department
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Table: Computation of Posterior Probabilities

(1) Outcome	(2) Prior Probabilities	(3) Conditional Probabilities	(4) Joint Probabilities (2) x (3)	(5) Posterior Probabilities (4) /∑(4)

BAYES' THEOREM (for calculation of Posterior Probabilities)

The probability of event B_i given that event A has occurred is given by the formula

$$P(B_i|A) = \frac{P(B_i)P(A|B_i)}{P(B_1)P(A|B_1)+P(B_2)P(A|B_2)+\cdots+P(B_k)P(A|B_k)}$$

where $B_1, B_2, \cdots, B_k$ are **mutually exclusive** and **collectively exhaustive** events.

Control Charts

***R* chart**	**$\bar{x}$ chart**
UCL = $\bar{R}\, D_4$ LCL = $\bar{R}\, D_3$ $where\ \bar{R} = \frac{\sum R}{k}$ k= number of sub groups sampled	UCL = $\bar{\bar{x}} + A_2\, \bar{R}$ LCL = $\bar{\bar{x}} - A_2 \bar{R}$ $where\ \bar{\bar{x}} = \frac{\sum \bar{x}}{k}$

***p* chart**	***c* chart**
Control Limits: $\bar{p} \pm 3\sqrt{\frac{(\bar{p})(\bar{q})}{\bar{n}}}$ where $\bar{p} = \frac{\sum x}{\sum n}$, $\bar{q} = 1 - \bar{p}$, $\bar{n} = \frac{\sum n}{k}$ where k = number of subgroups sampled	**Control Limits:** $\bar{c} \pm 3\sqrt{\bar{c}}$ where $\bar{c} = \frac{\sum c}{k}$ c = number of occurrences and k = number of units sampled

Table of Control Chart Constants

OBSERVATIONS IN SAMPLE, *n*	CHART FOR AVERAGES $\bar{x}$		CHART FOR RANGES $\bar{R}$	
	FACTORS FOR CONTROL LIMITS			
	A_2	A_3	D_3	D_4
2	1.880	2.659	0	3.267
3	1.023	1.954	0	2.575
4	0.729	1.628	0	2.282
5	0.577	1.427	0	2.114
6	0.483	1.287	0	2.004
7	0.419	1.182	0.076	1.924
8	0.373	1.099	0.136	1.864
9	0.337	1.032	0.184	1.816
10	0.308	0.975	0.223	1.777
11	0.285	0.927	0.256	1.744
12	0.266	0.886	0.283	1.717
13	0.249	0.850	0.307	1.693
14	0.235	0.817	0.328	1.672
15	0.223	0.789	0.347	1.653
16	0.212	0.763	0.363	1.637
17	0.203	0.739	0.378	1.622
18	0.194	0.718	0.391	1.609
19	0.187	0.698	0.404	1.596
20	0.180	0.680	0.415	1.585
21	0.173	0.663	0.425	1.575
22	0.167	0.647	0.435	1.565
23	0.162	0.633	0.443	1.557
24	0.157	0.619	0.452	1.548
25	0.153	0.606	0.459	1.541

Source: Adapted from *Manual on Presentation of Data and Control Chart Analysis, Copyright ©2002 by ASTM International, p. 67*

Forecasting

Moving Average

$$k\text{-}\ period\ moving\ average\ = \sum(actual\ value\ in\ previous\ k\ periods)/k$$

Weighted Moving Average

$$k\text{-}period\ weighted\ \ moving\ average\ = \frac{\sum_i^k(\text{weight for period } i) \times (\text{Actual value in period } i)}{\sum_i^k(\text{weights})}$$

Exponential Smoothing

$F_{t+1} = F_t + \alpha\,(A_t - F_t)$ or $F_{t+1} = \alpha A_t + (1-\alpha)F_t$

where A_t = actual value in period t , F_t = forecast for period t

F_{t+1} = forecast for period t +1,

α = smoothing constant value ($0 \le \alpha \le 1$)

$$MAD = \sum_{t=1}^{T} |\ forecast\, error\ | / T \qquad MAPE = 100 \sum_{t=1}^{T} \left(|\ forecast\, error\ | / A_t \right) / T$$

Simple Linear Regression

$\hat{y} = a + b\,x$ **or** $\hat{y} = b_0 + b_1 x$

***Coefficient of Correlation* (r)** measures the strength of the relationship ($-1 \le r \le 1$)

***Coefficient of Determination* (R^2)** = $\dfrac{\text{explained variation}}{\text{total variation}}$ or $\dfrac{SSR}{SST}$

where SST = SSR + SSE

SST = total sum of squares, SSR = regression sum of squares, SSE = error sum of squares

***Standard Error of Estimate* (s_e)** $s_e = \sqrt{\dfrac{SSE}{df}}$ or $s_e = \sqrt{\dfrac{SSE}{n-2}}$

Multiple Regression

$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \ldots + b_k x_k$

Standard Error of Estimate (s_e) $s_e = \sqrt{\dfrac{SSE}{df}}$ or $s_e = \sqrt{\dfrac{SSE}{n-(k+1)}}$ or $s_e = \sqrt{\dfrac{\sum (y-\hat{y})^2}{n-(k+1)}}$

where $\hat{y}$ = predicted y value and y = actual y value

df = n- (k + 1)

df =degrees of freedom, k = number of independent (x) variables, n= sample size

Coefficient of Determination (R^2) = $\dfrac{\text{explained variation}}{\text{total variation}}$ or $\dfrac{SSR}{SST}$

Adjusted R^2 = $1 - \dfrac{(n-1)}{[n-(k+1)]}(1-R^2)$

where n = sample size and k = number of independent (x) variables