CALC 203

FINAL EXAM FORMULA SHEET

- Assume u & v are functions of x.

➢ ***Differentiation***

$$\frac{d}{dx}\left[cu^n\right] = cnu^{n-1}\cdot u' \quad \frac{d}{dx}\left[\ln u\right] = \frac{u'}{u} \quad \frac{d}{dx}\left[\log_b u\right] = \frac{u'}{u\ln b} \quad \frac{d}{dx}\left[e^u\right] = e^u\cdot u' \quad \frac{d}{dx}\left[b^u\right] = b^u\cdot u'\cdot \ln b$$

$$\frac{d}{dx}\left[\sin u\right] = \cos u\cdot u' \quad \frac{d}{dx}\left[\cos u\right] = -\sin u\cdot u' \quad \frac{d}{dx}\left[\tan u\right] = \sec^2 u\cdot u' \quad \frac{d}{dx}\left[\cot u\right] = -\csc^2 u\cdot u'$$

$$\frac{d}{dx}\left[\sec u\right] = \sec u\tan u\cdot u' \quad \frac{d}{dx}\left[\csc u\right] = -\csc u\cot u\cdot u' \quad \frac{d}{dx}\left[uv\right] = u'v + uv' \quad \frac{d}{dx}\left[\frac{u}{v}\right] = \frac{u'v - uv'}{v^2}$$

➢ ***Integration***

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \ (n \neq -1) \qquad \int \frac{1}{u}\, du = \ln|u| + C \qquad \int e^u du = e^u + C$$

$$\int \sin u\ du = -\cos u + C \qquad \int \cos u\ du = \sin u + C \qquad \int \tan u\ du = -\ln|\cos u| + C$$

$$\int \cot u\ du = \ln|\sin u| + C \qquad \int \sec u\ du = \ln|\sec u + \tan u| + C \qquad \int \csc u\ du = \ln|\csc u - \cot u| + C$$

$$\int \sin^2 u\ du = \frac{u}{2} - \frac{\sin 2u}{4} + C \qquad \int \cos^2 u\ du = \frac{u}{2} + \frac{\sin 2u}{4} + C \qquad \int \tan^2 u\ du = \tan u - u + C$$

$$\int \cot^2 u\ du = -\cot u - u + C \qquad \int \sec^2 u\ du = \tan u + C \qquad \int \csc^2 u\ du = -\cot u + C$$

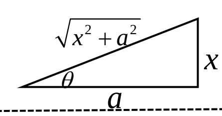
$$y_{avg} = \frac{1}{b-a}\int_a^b f(x)dx \qquad rms = \sqrt{\frac{1}{b-a}\int_a^b \left[f(x)\right]^2 dx}$$

By Parts $\int u\, dv = uv - \int v\, du$

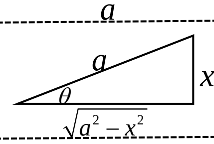
➢ ***Properties of Logarithm***: $\ln AB = \ln A + \ln B \qquad \ln\frac{A}{B} = \ln A - \ln B \qquad \ln A^P = P\ln A$

➢ ***Trignometric Substitutiion***:

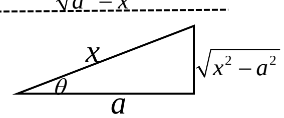
$\sqrt{x^2+a^2}$ Let $x = a\tan\theta$, $\sqrt{x^2+a^2} = a\sec\theta$



$\sqrt{a^2-x^2}$ Let $x = a\sin\theta$, $\sqrt{a^2-x^2} = a\cos\theta$



$\sqrt{x^2-a^2}$ Let $x = a\sec\theta$, $\sqrt{x^2-a^2} = a\tan\theta$



Compiled by: Humber College Math Department
Last revision: 06/2015

- ***Trignometric Identities*** :

$\sin^2\theta + \cos^2\theta = 1$ $\qquad 1 + \tan^2 = \sec^2\theta$

$\cot\theta = \frac{1}{tan\theta}$ $\qquad \sec\theta = \frac{1}{tan\theta}$ $\qquad \csc\theta = \frac{1}{sin\theta}$

- ***Solutions to Second-order DE with right side zero,*** $ay'' + by' + cy = 0$

The auxiliary equation has the form of $am^2 + bm + c = 0$ and:

Roots of the Auxiliary Equation	Solution to $ay'' + by' + cy = 0$
Real and Unequal (two real roots m_1 and m_2)	$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$
Real and Equal (double root *m*)	$y = c_1 e^{mx} + c_2 x e^{mx}$
Non Real (complex roots $A \pm Bi$)	$y = e^{Ax}(c_1 \cos Bx + c_2 \sin Bx)$

- ***Partial Sum Test:***

$$\lim_{n\to\infty} S_n = S \begin{cases} \text{If the limit exists, then the series converges.} \\ \text{If the limit does not exist, then the series diverges.} \end{cases}$$

Geometric Sequence: Sum of *n* terms $S_n = \frac{a(1-r^n)}{1-r} (a: \textit{first term}, \; r: \textit{common ratio})$

Sum to infinity $S_n = \frac{a}{1-r} \; \textit{where} \, |r| < 1 \; (a: \textit{first term}, \; r: \textit{common ratio})$

- ***Ratio Test:***

$$\lim_{n\to\infty} \left| \frac{u_{n+1}}{u_n} \right| \begin{cases} <1 & \text{converges} \\ >1 & \text{diverges} \\ =1 & \text{test fails} \end{cases}$$

- ***Maclaurin Series:***

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + ---- + \frac{f^{(n)}(0)}{n!}x^n + ---$$

- ***Fourier Series:***

$$f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + ---- + a_n \cos nx + ---$$
$$+ b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + ---- + b_n \sin nx + ---$$

where $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)dx \quad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$