TMTH 114

Final Exam Formula Sheet

Chapter 7: Right Triangles

$$1 \text{ rev} = 360^{\circ} = 2\pi \text{ rad}, \quad 1^{\circ} = 60', \quad 1' = 60'', \quad 1 \text{ rad} \approx 57.3^{\circ}$$

$$1^{\circ} = 60'$$
.

$$1' = 60$$
".

$$1 \text{ rad} \approx 57.3^{\circ}$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp'}}, \qquad \cos \theta = \frac{\text{adj}}{\text{hyp'}}, \qquad \tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$c^2 = a^2 + b^2$$
 (Pythagorean Theorem)

Given $(x, y) \neq (0,0)$ on terminal arm of angle θ , let $r = \sqrt{x^2 + y^2}$. Then:

$$\sin \theta = \frac{y}{r}$$

$$\sin \theta = \frac{y}{r}$$
 $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$

$$\tan \theta = \frac{y}{y}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$csc \theta = \frac{1}{\sin \theta}$$
 $sec \theta = \frac{1}{\cos \theta}$
 $cot \theta = \frac{1}{\tan \theta}$

$$\cot \theta = \frac{1}{\tan \theta}$$

Chapter 8: Factoring

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$
 $a^2 - b^2 = (a - b)(a + b)$

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$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

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 $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$

Chapter 13: Exponents and Radicals

$$\sqrt[n]{a} = a^{1/n}$$

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 $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

Given nonzero real numbers x and y, and integers m and n:

$$x^1=x$$

$$x^0 = 1$$

$$x^{-n} = \frac{1}{x^n}$$

$$(x^m)^n = x^{m \cdot n}$$

$$x^m \cdot x^n = x^{m+n}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$(xy)^n=x^ny^n$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$$

Chapter 14: Quadratic Equations

Given
$$ax^2 + bx + c = 0$$
, where $a \ne 0$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (quadratic formula)

Chapter 15: Oblique Triangles and Vectors

$$\sin \theta = \sin(180^{\circ} - \theta)$$
 $\cos \theta = \cos(360^{\circ} - \theta)$ $\tan \theta = \tan(180^{\circ} + \theta)$

Law of Sines:
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines:
$$a^2 = b^2 + c^2 - 2bc \cos A \qquad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$
 $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$$c^2 = a^2 + b^2 - 2ab \cos C$$
 $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Chapter 17: Trigonometric Functions

Sine wave as a function of an angle x: $y = a \sin(bx + c)$

amplitude =
$$|a|$$
 period = $\frac{360^{\circ}}{b}$ or $\frac{2\pi}{b}$ frequency = $\frac{b}{360^{\circ}}$ or $\frac{b}{2\pi}$

phase angle =
$$c$$
 phase shift = $-\frac{c}{b}$

Sine wave as a function of time t: $y = a \sin(\omega t + \phi)$

amplitude =
$$|a|$$
 angular velocity = ω period = $\frac{2\pi}{\omega}$

frequency =
$$\frac{\omega}{2\pi}$$
 phase angle = ϕ phase shift = $-\frac{\phi}{\omega}$

 $\cos \theta = \sin(\theta + 90^{\circ})$ Cosine and Sine Curves Related:

Sinusoidals as phasors:

$$a \sin(\omega t + \phi)$$
 is identified with $a \angle \phi$,

$$a \cos(\omega t + \phi)$$
 is identified with $a \angle (\phi + 90^{\circ})$

Chapter 18: Trigonometric Identities and Equations

$$csc \theta = \frac{1}{\sin \theta}, \qquad sec \theta = \frac{1}{\cos \theta}, \qquad cot \theta = \frac{1}{\tan \theta}$$

$$tan \theta = \frac{\sin \theta}{\cos \theta}, \qquad cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \qquad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$
, $\tan^2\theta + 1 = \sec^2\theta$, $1 + \cot^2\theta = \csc^2\theta$

Chapter 19: Ratio, Proportion, and Variation

Direct Variation:
$$y = kx$$
 or $\frac{y_2}{y_1} = \frac{x_2}{x_1}$

Power Variation:
$$y = kx^n$$
 or $\frac{y_2}{y_1} = \frac{(x_2)^n}{(x_1)^n}$

Inverse Variation:
$$y = \frac{k}{x}$$
 or $\frac{y_2}{y_1} = \frac{x_1}{x_2}$

Joint Variation:
$$y = kxw$$

Chapter 20: Exponential and Logarithmic Functions

Growth: Decay: Growth to an Upper Limit:

$$y = ae^{nt} y = ae^{-nt} y = a (1 - e^{-nt})$$

Exponential Form: $y = b^x$ Logarithmic Form: $\log_b y = x$

Properties of logarithms (where b, M, N > 0, b \neq 1, and p is a real number):

$$\log_b MN = \log_b M + \log_b N \qquad \qquad \log_b \frac{M}{N} = \log_b M - \log_b N$$

$$\log_b M^p = p \cdot \log_b M \qquad \qquad \log_b 1 = 0 \qquad \qquad \log_b b = 1$$

$$\log_b b^M = M$$
 $\log_b a = \frac{\log a}{\log b} = \frac{\ln a}{\ln b}$

Common logarithm: $\log x = \log_{10} x$

Natural logarithm: $\ln x = \log_e x$, where $e \approx 2.718$

Chapter 21: Complex Numbers

The Imaginary Unit and its Powers: $j = \sqrt{-1}$, $j^2 = -1$, $j^3 = -j$, $j^4 = 1$, $j^5 = j$, ...

Complex Number in Rectangular Form: x + jy x = real part, y = imaginary part

Complex Number in Polar Form: $r \angle \theta$ $r = \text{magnitude}, \theta = \text{polar angle}$

Polar to Rectangular Form: $r \angle \theta = r \cos \theta + j r \sin \theta$

$$x + j y = \sqrt{x^2 + y^2} \angle \tan^{-1} \left(\frac{y}{x}\right)$$

Complex current, voltage, and impedance:

Given
$$i = I_{max} \sin(\omega t + \phi)$$

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$$i = I_{max} \sin(\omega t + \phi)$$
, $I = I_{eff} \angle \phi$, where $I_{eff} = \frac{I_{max}}{\sqrt{2}}$

Given
$$v = V_{max} \sin(\omega t + \phi)$$
,

Given
$$v = V_{max} \sin(\omega t + \phi)$$
, $V = V_{eff} \angle \phi$, where $V_{eff} = \frac{V_{max}}{\sqrt{2}}$

$$Z = R + jX = \sqrt{R^2 + X^2} \, \angle \phi$$

$$Z=R+jX=\sqrt{R^2+X^2}\ \angle\phi$$
 , where $X=X_L-X_C$ and $\phi=\tan^{-1}\left(\frac{X}{R}\right)$

Ohm's Law for AC circuits: V = ZI

$$V = ZI$$

Chapter 22: Analytic Geometry

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$
, where (x_1, y_1) and (x_2, y_2) are two distinct points on the line

$$m = \tan \theta$$
, where θ is the line's angle of inclination

$$\theta = \begin{cases} \tan^{-1} m, & \text{if } m \ge 0 \\ \tan^{-1} m + 180^{\circ}, & \text{if } m < 0 \end{cases}, \text{ where } m \text{ is the line's slope}$$

Equation of Straight Line:

• General Form:
$$Ax + By + C = 0$$

• Slope-Intercept Form:
$$y = mx + b$$

• Point-Slope Form:
$$y - y_1 = m(x - x_1)$$

• Two-point Form:
$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$