TMTH 220
FINAL EXAM FORMULA SHEET

CHAPTER 5: *Graphs*

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} \qquad y = mx + b$$

CHAPTER 11: *Determinants*

Cramer's Rule:

a) For a system of two linear equations with two unknowns:

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \quad \text{and} \quad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

b) For a system of three linear equations with three unknowns:

$$x = \frac{\begin{vmatrix} k_1 & b_1 & c_1 \\ k_2 & b_2 & c_2 \\ k_3 & b_3 & c_3 \end{vmatrix}}{\Delta} \qquad y = \frac{\begin{vmatrix} a_1 & k_1 & c_1 \\ a_2 & k_2 & c_2 \\ a_3 & k_3 & c_3 \end{vmatrix}}{\Delta} \qquad z = \frac{\begin{vmatrix} a_1 & b_1 & k_1 \\ a_2 & b_2 & k_2 \\ a_3 & b_3 & k_3 \end{vmatrix}}{\Delta}$$

where $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$

CHAPTER 12: *Matrices*

Inverse matrix: $AA^{-1} = A^{-1}A = I$

Solution of the system of linear equations: $AX = B$ is $X = A^{-1}B$

Multiplication of matrices: $\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}\begin{pmatrix} x & u \\ y & v \\ z & w \end{pmatrix} = \begin{pmatrix} ax + by + cz & au + bv + cw \\ dx + ey + fz & du + ev + fw \end{pmatrix}$

CHAPTER 13: *Exponents and Radicals*

$$\sqrt[n]{a} = a^{\frac{1}{n}} \qquad (a \pm b)^2 = a^2 \pm 2ab + b^2$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m \qquad (a - b)(a + b) = a^2 - b^2$$

CHAPTER 14: *Quadratic Equations*

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

CHAPTER 15: *Oblique Triangles and Vectors*

$\sin\theta = \sin(180° - \theta)$ $\cos\theta = \cos(360° - \theta)$ $\tan\theta = \tan(180° + \theta)$

Law of Sines: $\dfrac{a}{\sin(A)} = \dfrac{b}{\sin(B)} = \dfrac{c}{\sin(C)}$

Law of Cosines:

$a^2 = b^2 + c^2 - 2bc\cos(A)$ $\cos(A) = \dfrac{b^2+c^2-a^2}{2bc}$

$b^2 = a^2 + c^2 - 2ac\cos(B)$ $\cos(B) = \dfrac{a^2+c^2-b^2}{2ac}$

$c^2 = a^2 + b^2 - 2ab\cos(C)$ $\cos(C) = \dfrac{a^2+b^2-c^2}{2ab}$

CHAPTER 17: *Graphs of the Trigonometric Functions*

General Sine Wave: $y = a\sin(bx + c)$

amplitude $= |a|$ period $= \dfrac{360°}{b}$ or $\dfrac{2\pi}{b}$ frequency $= \dfrac{b}{360°}$ or $\dfrac{b}{2\pi}$

phase angle $= c$ phase shift $= -\dfrac{c}{b}$ $\cos\theta = \sin(\theta + 90°)$

Sine wave as a function of time t: $y = a\sin(\omega t + \phi)$

amplitude $= |a|$ angular velocity $= \omega$ period $= \dfrac{2\pi}{\omega}$

frequency $= \dfrac{\omega}{2\pi}$ phase angle $= \phi$ phase shift $= -\dfrac{\phi}{\omega}$

Addition of a sine wave and cosine wave:

$A\sin\omega t + B\cos\omega t = R\sin(\omega t + \phi)$ where

$R = \sqrt{A^2 + B^2}$ and $\phi = \arctan\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{B}{A}\right)$

CHAPTER 18: ***Trigonometric Identities and Equations***

$$\cot\theta = \frac{1}{tan\theta} \qquad \sec\theta = \frac{1}{cos\theta} \qquad \csc\theta = \frac{1}{sin\theta} \qquad \tan\theta = \frac{sin\theta}{cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$\sin^2\theta + \cos^2\theta = 1$ $\qquad$ $1 + \tan^2 = \sec^2\theta$ $\qquad$ $1 + \cot^2 = \csc^2\theta$

Sum and Difference Identities:

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \pm \tan A\tan B}$$

Double-Angle Identities:

$\sin 2A = 2\ sinAcosA$

$\cos 2A = cos^2A - sin^2A$

$\cos 2A = 1 - 2\ sin^2A$

$\cos 2A = 2\ cos^2A - 1$

$\tan 2A = \frac{2\tan A}{1 - tan^2 A}$

Half-Angle Identities:

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}} \qquad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

Function Values of Special Angles:

θ		**sin** θ	**cos** θ	**tan** θ	**cot** θ	**sec** θ	**csc** θ
30 ^{0}	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$ or $\frac{2}{\sqrt{3}}$	2
45 ^{0}	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60 ^{0}	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$	2	$\frac{2\sqrt{3}}{3}$ or $\frac{2}{\sqrt{3}}$

<u>CHAPTER 21:</u> *Complex Numbers*

The Imaginary Unit: $j = \sqrt{-1};\ \ j^2 = -1;\ \ j^3 = -j;\ \ j^4 = 1;\ \ j^5 = j$

Complex Numbers in Polar Form:

$$a + jb = r\angle\theta \quad where \quad r = \sqrt{a^2 + b^2} \quad and \quad \theta = arctan\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{b}{a}\right)$$

Conversion from Polar to Rectangular Form:

a = r cos θ   b = r sin θ

Complex Numbers in Trigonometric Form:

$a + jb = r(\cos\theta + j\sin\theta) = r\angle\theta$