# TMTH 220
# MIDTERM EXAM FORMULA SHEET

## CHAPTER 15: *Oblique Triangles and Vectors*

$\sin\theta = \sin(180° - \theta)$ $\cos\theta = \cos(360° - \theta)$ $\tan\theta = \tan(180° + \theta)$

**Law of Sines:** $\dfrac{a}{\sin(A)} = \dfrac{b}{\sin(B)} = \dfrac{c}{\sin(C)}$

**Law of Cosines:** $a^2 = b^2 + c^2 - 2bc\cos(A)$ $\cos(\mathrm{A}) = \dfrac{b^2+c^2-a^2}{2bc}$

$b^2 = a^2 + c^2 - 2ac\cos(B)$ $\cos(\mathrm{B}) = \dfrac{a^2+c^2-b^2}{2ac}$

$c^2 = a^2 + b^2 - 2ab\cos(C)$ $\cos(\mathrm{C}) = \dfrac{a^2+b^2-c^2}{2ab}$

## CHAPTER 17: *Graphs of the Trigonometric Functions*

**General Sine Wave:** $y = \mathrm{a}\sin(bx + c)$

amplitude = $|\mathrm{a}|$ period = $\dfrac{360°}{b}$ or $\dfrac{2\pi}{b}$ frequency = $\dfrac{b}{360°}$ or $\dfrac{b}{2\pi}$

phase angle = $c$ phase shift = $-\dfrac{c}{b}$ $\cos\theta = \sin(\theta + 90°)$

**Sine wave as a function of time t:** $y = \mathrm{a}\sin(\omega t + \phi)$

amplitude = $|\mathrm{a}|$ angular velocity = $\omega$ period = $\dfrac{2\pi}{\omega}$

frequency = $\dfrac{\omega}{2\pi}$ phase angle = $\phi$ phase shift = $-\dfrac{\phi}{\omega}$

**Addition of a sine wave and cosine wave:**

$A\sin\omega t + B\cos\omega t = R\sin(\omega t + \phi)$ where

$R = \sqrt{A^2 + B^2}$ and $\phi = \arctan\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{B}{A}\right)$

Compiled by: Humber College Math Department
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## <u>CHAPTER 18</u>: *Trigonometric Identities and Equations*

$$\cot\theta = \frac{1}{tan\theta} \qquad \sec\theta = \frac{1}{cos\theta} \qquad \csc\theta = \frac{1}{sin\theta} \qquad \tan\theta = \frac{sin\theta}{cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$sin^2\theta + cos^2\theta = 1 \qquad 1 + tan^2 = sec^2\theta \qquad 1 + cot^2 = csc^2\theta$$

**Sum and Difference Identities:**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \pm \tan A \tan B}$$

**Double-Angle Identities:**

$$\sin 2A = 2\, sinAcosA \qquad \cos 2A = cos^2A - sin^2A \qquad \tan 2A = \frac{2\tan A}{1 - tan^2 A}$$

$$\cos 2A = 1 - 2\, sin^2A$$

$$\cos 2A = 2\, cos^2A - 1$$

**Half-Angle Identities:**

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}} \qquad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

***Function Values of Special Angles:***

| $\theta$ | | **sin** $\theta$ | **cos** $\theta$ | **tan** $\theta$ | **cot** $\theta$ | **sec** $\theta$ | **csc** $\theta$ |
|---|---|---|---|---|---|---|---|
| ***30***$^0$ | $\frac{\pi}{6}$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$ | $\sqrt{3}$ | $\frac{2\sqrt{3}}{3}$ or $\frac{2}{\sqrt{3}}$ | 2 |
| ***45***$^0$ | $\frac{\pi}{4}$ | $\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$ | 1 | 1 | $\sqrt{2}$ | $\sqrt{2}$ |
| ***60***$^0$ | $\frac{\pi}{3}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ | $\frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$ | 2 | $\frac{2\sqrt{3}}{3}$ or $\frac{2}{\sqrt{3}}$ |

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**<u>CHAPTER 21</u>:** ***Complex Numbers***

**The Imaginary Unit**: $j = \sqrt{-1};\ j^2 = -1;\ j^3 = -j;\ j^4 = 1;\ j^5 = j$

**Complex Numbers in Polar Form**:

$$a + jb = r\angle\theta \quad where \quad r = \sqrt{a^2 + b^2} \quad and \quad \theta = arctan\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{b}{a}\right)$$

**Conversion from Polar to Rectangular Form**:

a = r cosθ    b = r sinθ

**Complex Numbers in Trigonometric Form**:

$$a + jb = r(\cos\theta + j\sin\theta) = r\angle\theta$$